CORRECTION TO: HETEROCHAOS BAKER MAPS AND THE DYCK SYSTEM: MAXIMAL ENTROPY MEASURES AND A MECHANISM FOR THE BREAKDOWN OF ENTROPY APPROACHABILITY

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Lemma 3.3 and Corollary 3.4 in [2] are incorrect. We replace them by the following lemma.

Lemma 1. *For all* $a, b \in (0, \frac{1}{m})$ *we have* $\nu_\alpha(\Pi(\Lambda)) = \nu_\beta(\Pi(\Lambda)) = 1$.

We replace "From these equalities and Corollary 3.4, for all $a, b \in (0, \frac{1}{m})$ we obtain $\nu_\alpha(\Pi(\Lambda)) = \nu_\beta(\Pi(\Lambda)) = 1$" in the first paragraph of Section 3.7 by "By Lemma 1 we have $\nu_\alpha(A_\alpha \cap \Pi(\Lambda)) = 1$ and $\nu_\alpha(A_\beta \cap \Pi(\Lambda)) = 1$." The rest of the proof of Theorem 1.2 remains intact.

Proof of Lemma 1. Since $\Pi(\Lambda)$ is independent of (a, b), it suffices to show that

$$\nu_\alpha(\Pi_{\frac{1}{3},\frac{1}{6}}(\Lambda_{\frac{1}{3},\frac{1}{6}})) = 1 \text{ and } \nu_\beta(\Pi_{\frac{1}{6},\frac{1}{3}}(\Lambda_{\frac{1}{6},\frac{1}{3}})) = 1. \tag{1.1}$$

Let Leb denote the Lebesgue measure on $[0, 1]^3$. For the map $f = f_{\frac{1}{3},\frac{1}{6}}$, Leb is invariant and ergodic [1, Theorem 1.2]. Birkhoff's ergodic theorem implies $\mathrm{Leb} \circ \Pi^{-1}(A_\alpha) = 1$. By Lemma 3.1 we have $h(\sigma, \mathrm{Leb} \circ \Pi^{-1}) = h(f, \mathrm{Leb})$. Meanwhile, a direct calculation using Shannon-Mcmillan-Breiman's theorem shows $h(f, \mathrm{Leb}) \geq \log(m + 1)$, and hence $h(\sigma, \mathrm{Leb} \circ \Pi^{-1}) = \log(m + 1)$. Since Leb has negative central Lyapunov exponent with respect to f, we obtain $\nu_\alpha = \mathrm{Leb} \circ \Pi^{-1}$ and so $\nu_\alpha(\Pi(\Lambda)) = \mathrm{Leb}(\Lambda) = 1$ as required. A proof of the second equality in (1.1) is analogous. Alternatively, one can use the involution $\iota_D \colon \Sigma_D \to \Sigma_D$ introduced in the proof of Lemma 3.8: $\iota_D(\Pi_{\frac{1}{3},\frac{1}{6}}(\Lambda_{\frac{1}{3},\frac{1}{6}})) = \Pi_{\frac{1}{6},\frac{1}{3}}(\Lambda_{\frac{1}{6},\frac{1}{3}})$ and $\nu_\beta = \nu_\alpha \circ \iota_D^{-1}$. □

References

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2. Takahasi, H. and Yamamoto, K., Heterochaos baker maps and the Dyck system: maximal entropy measures and a mechanism for the breakdown of entropy approachability. *Proc. Amer. Math. Soc.*, to appear, doi:10.1090/proc/16538.

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